

University of California, Berkeley
Physics 110A, Section 2, Spring 2003 (*Strovink*)

PROBLEM SET 6

1.

A coaxial cable consists of an inner conductor of radius a and an outer conductor of inner and outer radii b and c respectively. A steady current I flows along \hat{z} in the inner conductor and along $-\hat{z}$ in the outer conductor. Because the current is steady, the volume current density is uniform in both conductors. Find the magnetic field everywhere.

2.

A toroid of inner radius a , outer radius b , height h , and rectangular cross section is wound with N finely pitched turns each carrying current I .

(a.)

Assuming that it lies along $\hat{\phi}$, find the magnetic field everywhere.

(b.)

The inductance L of this toroid is Φ_B/I , where Φ_B is N times the magnetic flux linked by one turn. Show that L is proportional to h , to $\ln b/a$, and to N^2 .

3.

At 45° latitude, the magnetic field of the earth is about half a gauss (0.5×10^{-4} T). At the earth's surface, the magnetic field shape is similar to that of an ideal dipole located at its center.

(a.)

Estimate the earth's magnetic dipole moment.

(b.)

If this moment were due to an iron ball fully magnetized to $\mu_0 M = 2$ T, what would be the ratio of the ball's radius to the earth's radius?

4.

A *quadrupole magnet* is one for which, within a limited region,

$$\vec{B} = k_0(\hat{x}y + \hat{y}x),$$

where k_0 is a constant. Charged particles pass through such a magnet nearly in the z direction, very close to the z axis.

(a.)

Show that $\nabla \cdot \vec{B} = 0$ and $\nabla \times \vec{B} = 0$. Therefore the quadrupole-field region can be evacuated – no current needs to flow within it.

(b.)

Show that if $k_0 > 0$ the quadrupole magnet focusses the x (or y) motion of the particles, but defocusses their y (or x) motion. Which pair of directions is correct?

(Because of their focussing/defocussing properties, *pairs* of quadrupole magnets must be used to achieve a (weaker) focus simultaneously in both the x and y directions. This technique was put to first practical use in the discovery of the antiproton at the Berkeley Bevatron in 1956.)

(c.)

In a *Panofsky quadrupole* (an unusual type), the field region lies within a volume of square cross section $|x| < b$ and $|y| < b$. Four current sheets are located at

$$(1) \quad x = b, \quad |y| < b$$

$$(2) \quad x = -b, \quad |y| < b$$

$$(3) \quad y = b, \quad |x| < b$$

$$(4) \quad y = -b, \quad |x| < b.$$

Just outside the current sheets, an iron yoke is placed. Because the permeability of the iron is very high, any field component just outside each current sheet that is parallel to the sheet (and the yoke face) must essentially vanish. What surface current densities \vec{K}_1 thru \vec{K}_4 must be supplied? (Wolfgang Panofsky, who in 1951 collided with U.C. Berkeley's loyalty oath, moved to Stanford and eventually founded SLAC.)

5.

In lecture we generalized the Biot-Savart law to consider a volume current density \vec{J} as the source of a magnetic field:

$$4\pi \frac{\vec{B}(\vec{r})}{\mu_0} = \int d\tau' \frac{\vec{J}(\vec{r}') \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3},$$

assuming that \vec{J} vanishes at ∞ and the volume integral extends over all space. We then showed that this equation is equivalent to Ampère's law

$$\nabla \times \frac{\vec{B}}{\mu_0} = \vec{J}.$$

Using (but not rederiving) the above mathematics, show that if \vec{B} vanishes at ∞ , the vector potential \vec{A} can be obtained by integrating \vec{B} over all space:

$$4\pi \vec{A}(\vec{r}) = \int d\tau' \frac{\vec{B}(\vec{r}') \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}.$$

6.+7.

When perfect cylindrical symmetry about the \hat{z} axis is present, and in addition nothing varies along z , we showed in class that a uniform magnetic field,

$$\vec{B}(\vec{r}) = B_0 \hat{z},$$

arises from the vector potential

$$\vec{A}(\vec{r}) = \frac{1}{2} s B_0 \hat{\phi},$$

where s and ϕ are the usual cylindrical coordinates.

(a.)

At $t = 0$ a nonrelativistic particle of mass m and charge e is born at the origin, with initial momentum

$$\vec{p}(t=0) = \hat{y} p_0.$$

Determine its trajectory through the magnetic field.

(b.)

Pick a couple of points on the trajectory in addition to the origin. Show that

$$\vec{r} \times (\vec{p} + e\vec{A}) = 0$$

at all three points.

[This is no accident. The *canonical momentum* \vec{P} for a free particle in a static electromagnetic field is

$$\vec{P} = \vec{p} + e\vec{A}.$$

The cylindrical symmetry of the problem about the z axis requires the z component of the canonical angular momentum $\vec{r} \times \vec{P}$ to be conserved.]

(c.)

The late Herbert L. Anderson (famous for building the Chicago Cyclotron and holding Fermi's notebook) is reputed always to have asked the same graduate student oral exam problem:

"Consider a cyclotron magnet, cylindrically symmetric about the z axis, whose pair of circular pole faces are located symmetrically about the origin in planes of constant z . Outside the pole faces, in those regions accessible to particles, the field remains essentially cylindrically symmetric; it drops off steeply with s so that both \vec{B} and \vec{A} become negligible for $s > s_0$. Neglect gravity. From outside s_0 , a charged particle is launched toward the origin along the x axis. After being bent by the magnetic field, eventually the particle finds itself again outside s_0 . At that time, from what point does the bent particle appear to emanate? Prove your assertion."

Try to pass Anderson's exam.

8.

Calculate the magnetic flux

$$\Phi_B \equiv \iint \vec{B} \cdot d\vec{a}$$

through a disk of radius b , coaxial with \hat{z} , centered a distance z above an ideal magnetic dipole of moment $\vec{m} = \hat{z}m$.

[Hint: Use the vector potential to avoid a messy integral.] (Strangely, taking advantage of the proportionality of ideal electric and magnetic dipole fields away from the origin, this is the easiest way to calculate the *electric* flux from an *electric* dipole through the same disk.)